2.1

Shape and Structure

Forms of Quadratic Functions

LEARNING GOALS

In this lesson, you will:

- Match a quadratic function with its corresponding graph.
- Identify key characteristics of quadratic functions based on the form of the function.
- Analyze the different forms of quadratic functions.
- Use key characteristics of specific forms of quadratic functions to write equations.
- Write quadratic functions to represent problem situations.

KEY TERMS

- standard form of a quadratic function
- factored form of a quadratic function
- vertex form of a quadratic function
- concavity of a parabola

Have you ever seen a tightrope walker? If you've ever seen this, you know that it is quite amazing to witness a person able to walk on a thin piece of rope. However, since safety is always a concern, there is usually a net just in case of a fall.

That brings us to a young French daredevil named Phillippe Petit. Back in 1974 with the help of some friends, he spent all night secretly placing a 450 pound cable between the World Trade Center Towers in New York City. At dawn, to the shock and amazement of onlookers, the fatigued 24-year old Petit stepped out onto the wire. Ignoring the frantic calls of the police, he walked, jumped, laughed, and even performed a dance routine on the wire for nearly an hour without a safety net! Mr. Petit was of course arrested upon climbing back to the safety of the ledge. When asked why he performed such an unwise, dangerous act, Phillippe said: "When I see three oranges, I juggle; when I see two towers, I walk."

Have you ever challenged yourself to do something difficult just to see if you could do it?

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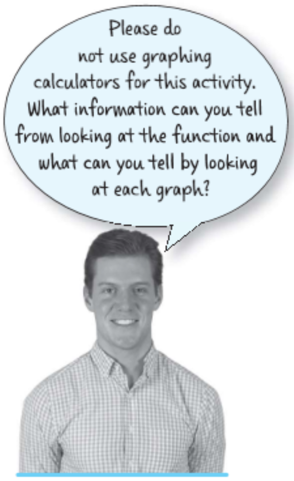
PROBLEM 1 It's All in the Form

1. Cut out each quadratic function and graph on the next page two pages.

   a. Tape each quadratic function to its corresponding graph.



   b. Explain the method(s) you used to match the functions with their graphs.

a. $f(x) = 2(x + 1)(x + 5)$	**d.** $f(x) = (x - 1)^2$	**g.** $f(x) = -(x + 4)^2 - 2$
b. $f(x) = \frac{1}{3}x^2 + \pi x + 6.4$	**e.** $f(x) = 2(x - 1)(x - 5)$	**h.** $f(x) = -5x^2 - x + 21$
c. $f(x) = -2.5(x - 3)(x - 3)$	**f.** $f(x) = x^2 + 12x - 1$	**i.** $f(x) = -(x + 2)^2 - 4$

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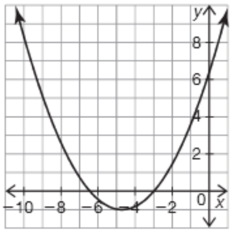
A.



tape function here

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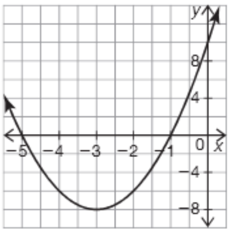

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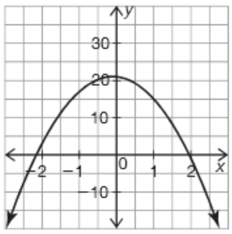

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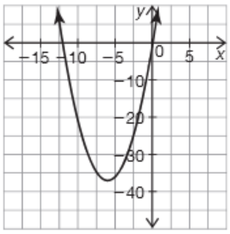
D.



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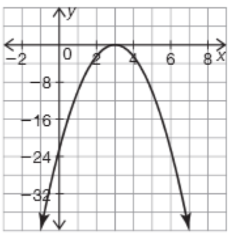
E.



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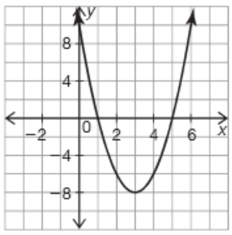
F.



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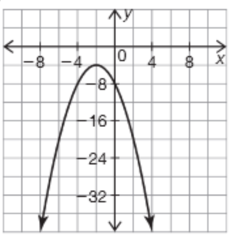
G.



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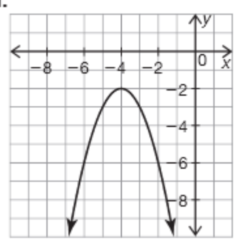
H.



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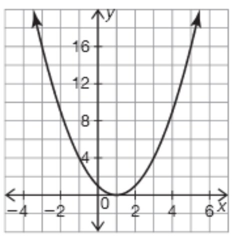
I.



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Recall that quadratic functions can be written in different forms.

- **standard form:** $f(x) = ax^2 + bx + c$, where a does not equal 0.
- **factored form:** $f(x) = a(x - r_1)(x - r_2)$, where a does not equal 0.
- **vertex form:** $f(x) = a(x - h)^2 + k$, where a does not equal 0.

2. Sort your graphs with matching equations into 3 piles based on the function form.

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The graphs of quadratic functions can be described using key characteristics:

- x-intercept(s),
- y-intercept,
- vertex,
- axis of symmetry, and
- concave up or down.

Concavity of a parabola describes whether a parabola opens up or opens down. A parabola is concave down if it opens downward; a parabola is concave up if it opens upward.

3. The form of a quadratic function highlights different key characteristics. State the characteristics you can determine from each.
   - standard form
   - factored form
   - vertex form

4. Christine and Kate were asked to determine the vertex of two different quadratic functions each written in different forms. Analyze their calculations.

Christine

$f(x) = 2x^2 + 12x + 10$

The quadratic function is in standard form. So I know the axis of symmetry is $x = \frac{-b}{2a}$.

$$x = \frac{-12}{2(2)}$$
$$= -3.$$

Now that I know the axis of symmetry, I can substitute that value into the function to determine the y-coordinate of the vertex.

$$f(-3) = 2(-3)^2 + 12(-3) + 10$$
$$= 2(9) - 36 + 10$$
$$= 18 - 36 + 10$$
$$= -8$$

Therefore, the vertex is $(-3, -8)$.

Kate

$g(x) = \frac{1}{2}(x + 3)(x - 7)$

The form of the function tells me the x-intercepts are -3 and 7. I also know the x-coordinate of the vertex will be directly in the middle of the x-intercepts. So, all I have to do is calculate the average.

$$x = \frac{-3 + 7}{2}$$
$$= \frac{4}{2} = 2$$

Now that I know the x-coordinate of the vertex, I can substitute that value into the function to determine the y-coordinate.

$$g(2) = \frac{1}{2}(2 + 3)(2 - 7)$$
$$= \frac{1}{2}(5)(-5)$$
$$= -12.5$$

Therefore, the vertex is $(2, -12.5)$.

a. How are these methods similar? How are they different?

b. What must Kate do to use Christine's method?

c. What must Christine do to use Kate's method?

5. Analyze each table on the following three pages. Paste each function and its corresponding graph from Question 2 in the "Graphs and Their Functions" section of the appropriate table. Then, explain how you can determine each key characteristic based on the form of the given function.

<table>
<tr><th colspan="6">Standard Form
$f(x) = ax^2 + bx + c$, where $a \neq 0$</th></tr>
<tr><th colspan="6">Graphs and Their Functions</th></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><th colspan="6">Methods to Identify and Determine Key Characteristics</th></tr>
<tr><td colspan="2">Axis of Symmetry</td><td colspan="2">x-intercept(s)</td><td colspan="2">Concavity</td></tr>
<tr><td colspan="3">Vertex</td><td colspan="3">y-intercept</td></tr>
</table>

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Factored Form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$		
Graphs and Their Functions		
Methods to Identify and Determine Key Characteristics		
Axis of Symmetry	x-intercept(s)	Concavity

Vertex	y-intercept

<table>
<tr><th colspan="6">Vertex Form
$f(x) = a(x - h)^2 + k$, where $a \neq 0$</th></tr>
<tr><th colspan="6">Graphs and Their Functions</th></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><th colspan="6">Methods to Identify and Determine Key Characteristics</th></tr>
<tr><td colspan="2">Axis of Symmetry</td><td colspan="2">x-intercept(s)</td><td colspan="2">Concavity</td></tr>
<tr><td colspan="3">Vertex</td><td colspan="3">y-intercept</td></tr>
</table>

PROBLEM 2 What Do You Know?

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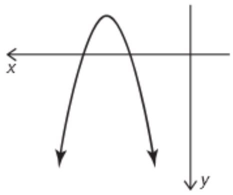
1. Analyze each graph. Then, circle the function(s) which could model the graph. Describe the reasoning you used to either eliminate or choose each function.

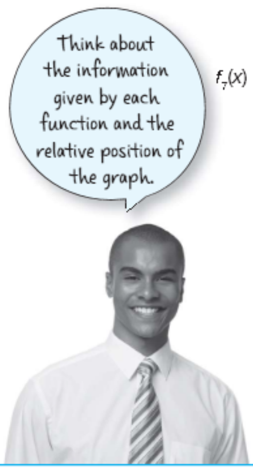
a.



$f_1(x) = -2(x + 1)(x + 4)$ $f_2(x) = -\frac{1}{3}x^2 - 3x - 6$ $f_3(x) = 2(x + 1)(x + 4)$

$f_4(x) = 2x^2 - 8.9$ $f_5(x) = 2(x - 1)(x - 4)$ $f_6(x) = -(x - 6)^2 + 3$

$f_7(x) = -3(x + 2)(x - 3)$ $f_8(x) = -(x + 6)^2 + 3$



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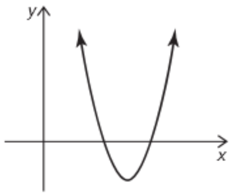

$f_1(x) = 2(x - 75)^2 - 92$ $f_2(x) = (x - 8)(x + 2)$ $f_3(x) = 8x^2 - 88x + 240$

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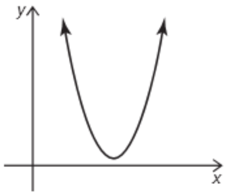
$f_4(x) = -3(x - 1)(x - 5)$ $f_5(x) = -2(x - 75)^2 - 92$ $f_6(x) = x^2 + 6x - 2$

$f_7(x) = 2(x + 4)^2 - 2$ $f_8(x) = (x + 1)(x + 3)$

c.



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$f_1(x) = 3(x + 1)(x - 5)$ $f_2(x) = 2(x + 6)^2 - 5$ $f_3(x) = 4x^2 - 400x + 10{,}010$

$f_4(x) = 3(x + 1)(x + 5)$ $f_5(x) = 2(x - 6)^2 + 5$ $f_6(x) = x^2 + 2x - 5$

2. Consider the two functions shown from Question 1. Identify the form of the function given, and then write the function in the other two forms, if possible. If it is not possible, explain why.

a. From part (a): $f_1(x) = -2(x + 1)(x + 4)$

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b. From part (c): $f_5(x) = 2(x - 6)^2 + 5$

PROBLEM 3 Unique . . . One and Only

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1. George and Pat were each asked to write a quadratic equation with a vertex of (4, 8). Analyze each student's work. Describe the similarities and differences in their equations and determine who is correct.

George	Pat
$y = a(x - h)^2 + k$	$y = a(x - h)^2 + k$
$y = a(x - 4)^2 + 8$	$y = a(x - 4)^2 + 8$
$y = -\frac{1}{2}(x - 4)^2 + 8$	$y = (x - 4)^2 + 8$

2. Consider the 3 forms of quadratic functions and state the number of unknown values in each.

Form	Number of Unknown Values
$f(x) = a(x - h)^2 + k$	
$f(x) = a(x - r_1)(x - r_2)$	
$f(x) = ax^2 + bx + c$	

a. If a function is written in vertex form and you know the vertex, what is still unknown?

b. If a function is written in factored form and you know the roots, what is still unknown?

c. If a function is written in any form and you know one point, what is still unknown? State the unknown values for each form of a quadratic function.

d. If you only know the vertex, what more do you need to write a unique function? Explain your reasoning.

e. If you only know the roots, what more do you need to write a unique function? Explain your reasoning.

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You can write a unique quadratic function given a vertex and a point on the parabola.

Write the quadratic function given the vertex (5, 2) and the point (4, 9).

Substitute the given values into the vertex form of the function.	$y = a(x - h)^2 + k$ $9 = a(4 - 5)^2 + 2$
Then simplify.	$9 = a(-1)^2 + 2$ $9 = 1a + 2$ $7 = 1a$ $7 = a$
Finally, substitute the *a*-value into the function.	$f(x) = 7(x - 5)^2 + 2$

You can write a unique quadratic function given the roots and a point on the parabola.

Write a quadratic function given the roots (−2, 0) and (4, 0), and the point (1, 6).

Substitute the given values into the factored form of the function.	$f(x) = a(x - r_1)(x - r_2)$ $6 = a(1 - (-2))(1 - 4)$
Then simplify.	$6 = a(1 + 2)(1 - 4)$ $6 = a(3)(-3)$ $6 = -9a$ $-\frac{2}{3} = a$
Finally, substitute the *a*-value into the function.	$f(x) = -\frac{2}{3}(x + 2)(x - 4)$

3. Explain why knowing the vertex and a point creates a unique quadratic function.

4. If you are given the roots, how many unique quadratic functions can you write? Explain your reasoning.

2

5. Use the given information to determine the most efficient form you could use to write the function. Write standard form, factored form, vertex form, or none in the space provided.

a. minimum point (6, −75)
y-intercept (0, 15) ________________

b. points (2, 0), (8, 0), and (4, 6) ________________

c. points (100, 75), (450, 75), and (150, 95) ________________

d. points (3, 3), (4, 3), and (5, 3) ________________

e. x-intercepts: (7.9, 0) and (−7.9, 0)
point (−4, −4) ________________

f. roots: (3, 0) and (12, 0)
point (10, 2) ________________

g. Max hits a baseball off a tee that is 3 feet high. The ball reaches a maximum height of 20 feet when it is 15 feet from the tee. ________________

h. A grasshopper was standing on the 35 yard line of a football field. He jumped, and landed on the 38 yard line. At the 36 yard line he was 8 inches in the air. ________________

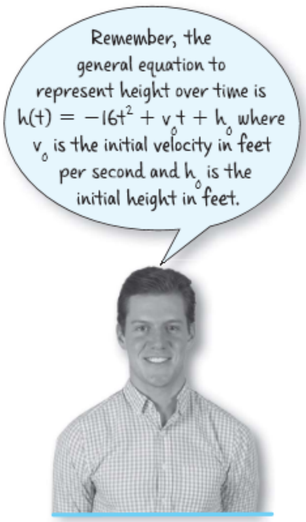
PROBLEM 4 Just Another Day at the Circus

Write a quadratic function to represent each situation using the given information. Be sure to define your variables.

1. The Amazing Larry is a human cannonball. He would like to reach a maximum height of 30 feet during his next launch. Based on Amazing Larry's previous launches, his assistant DaJuan has estimated that this will occur when he is 40 feet from the cannon. When Amazing Larry is shot from the cannon, he is 10 feet above the ground. Write a function to represent Amazing Larry's height in terms of his distance.

2. Crazy Cornelius is a fire jumper. He is attempting to run and jump through a ring of fire. He runs for 10 feet. Then, he begins his jump just 4 feet from the fire and lands on the other side 3 feet from the fire ring. When Cornelius was 1 foot from the fire ring at the beginning of his jump, he was 3.5 feet in the air. Write a function to represent Crazy Cornelius' height in terms of his distance. Round to the nearest hundredth.

3. Harsh Knarsh is attempting to jump across an alligator filled swamp. She takes off from a ramp 30 feet high with a speed of 95 feet per second. Write a function to represent Harsh Knarsh's height in terms of time.



Be prepared to share your solutions and methods.